SURVEY

MATHEMATICAL MODELING OF HEAT-EXCHANGE PROCESSES IN THE HUMAN BODY

G. N. Dul'nev, M. M. Korotkevich,
N. V. Pilipenko, and A. V. Sigalov

UDC 536.24

Methods of mathematical modeling of heat-exchange processes in the human body are used in various problems in the fields of medicine, physiology, athletics, the garment industry, and in the design of survival systems. Mathematical models describing the thermal states of the human body as a whole or its separate organs have been worked out in a large number of papers, starting from the 1940s. With the development of modern computational techniques and the more detailed physiological information available in recent years, the mathematical models have become more refined and more complex. With the help of numerical methods, complex models taking into account the anatomical structure of the body, different heat-transport mechanisms, and the effect of the thermoregulatory system can be worked out. In the present paper, we review the literature, discuss methods of constructing mathematical models, and present the application of the multistage modeling method in the analysis of the temperature field in the human body.

There are two main approaches in constructing mathematical models of the human body; the lumped parameter models and the distributed parameter models [1-3].

The lumped parameter models are represented by systems of ordinary differential equations for the average temperatures of different regions of the body, which are obtained from the heat-balance equations for each region. In distributed parameter models, partial differential equations describing the spatial temperature distribution are used.

It will be convenient to begin the review of the literature with a discussion of the lumped and distributed parameter models in general. Then using these two approaches, we describe the features of other widely used models.

Lumped Parameter Models. In this approach, the human body is divided into N elements. The thermal state of each element will be characterized by the volume-averaged temperatures of the tissues T_i, and blood (T_{ai}, T_{vi}) in arteries and veins.

We consider the derivation of the equations of the lumped parameter model, emphasizing the methods of obtaining the average temperatures, the assumptions made, and the physical interpretation of the parameters. Let an element of the system occupy a volume V_i, where part of its surface S_i' is in contact with other elements, and part S_i'' borders on the external medium. Arterial and venous blood flows into element i from neighboring elements. Part of the arterial blood flows through the capillary network of element i and is transformed into venous blood.

The change in the tissue enthalpy is given by the sum of the conductive heat fluxes Q_{ij} from neighboring elements, the heat flux from the external medium Q_{mi}, the fluxes Q_{ai} and Q_{vi} resulting from heat exchange with the arterial and venous blood, the heat flux Q_{ki} transmitted to the capillary blood, and the heat Q_{Mi} liberated from exchange processes [1, 4-6].

$$C_i \frac{dT_i}{d\tau} = Q_{ij} + Q_{\mathrm{m}} + Q_{ai} + Q_{vi} + Q_{vi} + Q_{Mi}. \tag{1}$$

The adequacy of model (1) will be determined by the accuracy of the approximate expressions coupling the thermal fluxes in (1) with the averaged tissue and blood temperatures (T_i, T_{ai}, T_{vi}) used as the input.

The conductive heat flux between elements i and j, passing through their common boundary, is normally given in terms of the thermal conductivity tensor σ_{ij} [1, 7, 8]:

Leningrad Institute of Exact Mechanics and Optics. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 46, No. 1, pp. 150-160, January, 1984. Original article submitted January 25, 1983.

0022-0841/84/4601-0117$08.50 © 1984 Plenum Publishing Corporation

$$Q_{ij} = \sigma_{ij}(T_j - T_i). \quad (2)$$

Note that the thermal conductivity defined in (2) depends on the temperature distribution in the body, i.e., it is a system parameter. This complication is tacitly avoided in the literature, where σ_{ij} is calculated from formulas for plane or cylindrical walls without taking into account blood flow and internal heat sources.

In determining the heat transport to the external medium, besides convection and radiation one must take into account the component of the heat flux Q_{ci} caused by evaporation from the surface of the skin and the respiratory tracks; the latter can vary under the action of the thermoregulatory system.

The heat fluxes transmitted by arterial and venous blood are given by the expressions [1, 2, 9]:

$$Q_{ai} = \sigma_{ai}(T_{ai} - T_i), \quad T_{vi} = \sigma_{vi}(T_{vi} - T_i). \quad (3)$$

The basic difficulty in describing heat-exchange processes with the arterial and venous blood is the determination of the thermal conductivities σ_{ai} and σ_{vi}. In order to determine these parameters quantitatively, one uses a model representation of the vascular system of the portion of the body under consideration, and considers the heat-exchange problem for this model system. Because the volume-averaged temperature T_i of the tissue appears in (3), and not the temperature averaged over the walls of the blood vessels, it is necessary in the calculation of the heat conductivities to take into account boundary layer drag in the blood vessels, as well as drag effects of the tissue between different blood vessels.

A theoretical analysis of the heat transport in blood vessels was given in [10] in which the velocity and temperature distributions were calculated for different boundary conditions on the blood vessel walls and the local Nusselt number dependence was obtained. In [11] the heat exchange between blood vessels and the adjacent tissue was considered for three cases: a single blood vessel, two parallel blood vessels with opposite flows, and a single vessel near the skin. However, these results still do not permit a correct theoretical determination of the conductivities σ_{ai} and σ_{vi}.

In writing an expression for the heat flux Q_{ki} transmitted in the capillary system [1-9], the assumption is made that heat exchange in the capillary system is ideal. It is then assumed that the blood entering the capillary has temperature T_{ai}, and after passing through the capillary takes on temperature T_i of the tissue. The resulting expression for Q_{ki} is

$$Q_{\kappa i} = G_{\kappa i} c_{kb}(T_{ai} - T_i). \quad (4)$$

Substituting the above expressions for the heat fluxes into the heat-balance equation (1), we obtain

$$C_i \frac{dT_i}{d\tau} = \sum_j \sigma_{ij}(T_j - T_i) + \sigma_{ci}(T_c - T_i) + Q_{ei} + \sigma_{ai}(T_{ai} - T_i) + \sigma_{vi}(T_{vi} - T_i) + G_{\kappa i} c_{kb}(T_{ai} - T_i) + Q_{Mi}. \quad (5)$$

The heat-balance equation for arterial blood in the i-th element can be written in the form [1-5, 9]

$$c_{kb} m_{ai} \frac{dT_{ai}}{d\tau} = \sigma_{ai}(T_i - T_{ai}) + \sigma_{av}(T_{vi} - T_{ai}) + c_{kb}[G_{ai}^{in} T_{ai}^{in} - G_{\kappa i} T_{ai} - (G_{ai}^{in} - G_{\kappa i}) T_{ai}^{out}], \quad (6)$$

and the corresponding equation for the venous blood is similar, where we use the fact that the venous blood leaving the i-th element is a combination of the venous blood entering the other elements, along with the blood passing through the capillary system with temperature T_i:

$$c_{kb} m_{vi} \frac{dT_{vi}}{d\tau} = \sigma_{vi}(T_i - T_{vi}) + \sigma_{av}(T_{ai} - T_{vi}) + c_{kb}[G_{vi}^{in} T_{vi}^{in} + G_{\kappa i} T_i - (G_{vi}^{in} + G_{\kappa i}) T_{vi}^{out}]. \quad (7)$$

The mean flow rate temperatures of the incoming arterial and venous flows (T_{ai}^{in}, T_{vi}^{in}) are calculated by averaging the temperatures of the flows from the j-th element to the i-th:

$$T_{ni}^{in} = \frac{1}{G_{ni}^{in}} \sum_j G_{nij} T_{nj}^{out}, \quad G_{ni}^{in} = \sum_j G_{nij}, \quad n = a, v. \tag{8}$$

In order to close the system of equations (5)-(8) it is necessary to add a relation between the blood temperatures T_{ni}, T_{ni}^{in}, and T_{ni}^{out}. This can be done by introducing a variation factor for the blood flows

$$\Psi_{ni} = (T_{ni} - T_{ni}^{in})/(T_{ni}^{out} - T_{ni}^{in}), \quad n = a, v.$$

The quantities Ψ_{ai} and Ψ_{vi} are determined by adopting a model for the vascular system of element i and calculating the change in blood temperature for motion along the blood vessels. However, this is a separate (and complicated) problem. In the literature the nonuniformity of the blood temperature field is not considered. Instead, it is assumed in [1-9] that $T_{ni}^{out} = T_{ni}$, n = a, v (the ideal mixing model).

The system of equations (5)-(8) represents a generalized model of a biological system with lumped parameters. A program for this model was developed in [12] for a system where the number of elements, heat couplings, and fluxes can be chosen arbitrarily. The structure of the specific system is taken as input information, given in extremely compact form. Therefore, the program makes it possible to calculate results rising different lumped parameter models.

Distributed Parameter Models. The human body is represented as a complex nonuniform system, consisting of different types of tissue permeated by a network of blood vessels of different diameters and distributed in space in a complicated way. In the calculation of the spatial temperature field in biological systems, a quasihomogeneous model of the body is used in which the thermal state is described with the help of locally uniform temperature fields for the tissues and blood. The locally averaged temperature T(x) at the point x is understood to mean an average over a volume ΔV around point x which is small in comparison to the size of the system, and yet is larger than the size of the nonuniformities.

The mathematical model of a quasihomogeneous body will consist of a system of differential equations which are derived assuming that the heat exchange between tissue and blood occurs at every point of the volume and can be characterized by a density of heat sources (sinks). The heat sources are specified with the help of the introduction of local volumetric coefficients of heat transport to the arterial α_a and venous α_v blood. For the heat exchange with blood passing through the capillaries, it is assumed that each point of the arterial blood passing through the capillaries (with mass flow rate per unit volume G) takes the local tissue temperature T(x). Then the differential equation describing heat transport in the tissues can be written in the form [1-3, 9, 13-16]:

$$c\rho \frac{\partial T}{\partial \tau} = \nabla(\lambda \nabla T) - G c_{kb} (T - T_a) - \alpha_a (T - T_a) - \alpha_v (T - T_v) + q_M. \tag{9}$$

In addition to (9), a mathematical model of the distributed parameter system must include equations describing heat transport by arterial and venous blood. However, we have not seen any treatments in the literature which consider spatial temperature distributions for the blood $T_a(x)$ and $T_v(x)$. Instead, volume-averaged blood temperatures T_{ai} and T_{vi} are used in (9). These temperatures are determined by considering balance equations of the type (6) or (7) in which the heat flux from the tissues to the blood is given by [1, 9, 16]:

$$Q_{ni} = \int_{V_i} \alpha_n [T(x) - T_{ni}]\, dV, \quad n = a, v. \tag{10}$$

In most papers it is assumed that the heat exchange in arteries and veins can be ignored in comparison to the capillary heat exchange [2-8, 14, 15]. Until entering the capillary, the arterial blood at any point of the body is assumed to have the temperature of blood leaving the heart. After passing through the capillaries the blood does not participate in heat exchange as it moves along the veins, therefore the temperature of blood feeding into a small circulation loop is equal to the mean flow rate temperature of blood leaving the capillary. With these assumptions, the mathematical model simplifies and we have

$$c\rho \frac{\partial T}{\partial \tau} = \nabla(\lambda \nabla T) - G c_{kb} (T - T_a) + q_M, \tag{11}$$

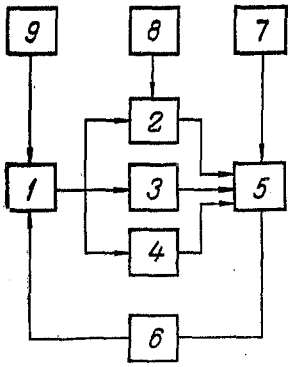

Fig. 1. Schematic diagram of the thermoregulatory system: 1) integrator; 2) heat production, 3) evaporation; 4) blood flow, 5) body; 6) heat receptors; 7) external medium; 8) work; 9) set point.

The temperature T_a is determined from the heat-balance equation for the heart—lung system [2-7, 14, 15] with account of the heat loss due to respiration (flux Q_r):

$$G_\Sigma c_{kb} T_a = \sum_i \int_{V_i} G c_{kb} T(\mathbf{x})\, dV - Q_r. \tag{12}$$

Here the summation goes over all elements V_i in which (11) is to be solved.

Thus, we see that the distributed parameter models used in the literature include the heat-conduction equation for the tissue (9) or (11) and integral forms of the heat-balance equations (6), (7), (10), or (12) for the blood. In treatments where heat exchange is considered in separate parts of the body without an analysis of the entire systems, the blood temperature T_a is given a priori.

Thermoregulatory Models. Modern computational techniques have been developed to the point where we can consider in principle numerical solutions of quite complicated heat-exchange models in humans. However, the basic difficulty is the modeling of the thermoregulatory system, which can change the parameters effecting the thermal state of the body (blood flow rate, magnitude of heat liberated, rate of evaporation of moisture). Therefore, in recent papers there has been continued interest in modeling the thermoregulatory reaction of the human system under various conditions, and in the adequate description of feedback in the thermoregulatory system [2-7, 14-23].

A schematic diagram of a possible model [15] of the thermoregulatory system in the human body is shown in Fig. 1. The body is subjecting to stimulation from the outside environment and release of heat from physical activity. The stimulation causes changes in the controlled quantities such as the temperatures and heat fluxes of different parts of the body which are picked up by the thermoreceptors. The observed quantities are compared with the reference values (the concept of set point) and the magnitude of the discrepancies determine the action on the system, which is directed to decreasing the discrepancies. Examples are changes in heat production (shivering), the intensity of diaphoresis and vasomotor reactions (constriction and expansion of veins).

In the literature there has been no unified treatment of the role of the thermoreceptors in the hypothalmus, skin, and other parts of the body in thermoregulation [17-21]. It is probable that this situation is due to the extremely large number of different empirical results describing feedback in the system. Feedback equations describe the coupling between the deviations of certain characteristic temperatures of the body from their reference values and the magnitude of the thermoregulatory action on the system. Many papers have considered the modeling of the thermal system in man under changing external conditions, and the development of adequate feedback equations by comparing the calculated results with experiment [6, 7, 14, 19, 20, 22]. In most papers, controlled parameters are taken to be the temperature of the brain and the weighted mean temperatures of different parts of the surface of the body. The most detailed look at the equations of thermoregulation with numerical data necessary for calculations is given in [22].

Some Models and Their Application. An historical review of the development of heat-exchange models in man is given in [1, 2]. Extensive bibliographies are also given in [3, 6, 14, 16].

Among the lumped parameter models, the most well known is that of Stolwijk [4, 5]. The initial model included the following elements: head (brain and scalp), trunk (bones, muscle, skin), extremities (muscle, skin), and the heart—lung system in which displacement of blood flow occurs. Further detail was added by using separate elements to model the arms, hands, legs, feet, and their internal structure (core region, muscle, fat, skin). This model was used to study the thermoregulatory system behavior in hypothermia under conditions of heavy physical work and electromagnetic radiation [5, 6, 22]. The model accounts for heat transport by conduction between adjacent layers; the separate parts of the body are coupled together by the blood flows and only capillary heat exchange is taken into account.

In [24] the Stolwijk model was used to calculate the thermal state of a human body in heated clothing.

Lumped parameter models of similar structure were used in a series of papers by Ermakova [7, 19, 20] to study thermoregulatory mechanisms.

In papers by Korobko [25, 26], lumped parameter models were used to describe the dynamics of the temperature field in artificial hyperthermia (radiation therapy in cancer treatment) with the goal of obtaining the optimal heating regime. From the specific heat-exchange conditions (immersion of the body in water) the following elements of the model were separated out: brain, scalp, neck, core region of body, skin, and heart—lung system.

In [8] the dynamical indentification of the lumped parameter model quantities (thermal conductivities, blood flow rates) was attempted by organized experimental research.

A large number of distributed parameter models have been proposed by Wissler [1, 9, 16]. The most complete model consists of 15 cylinders. The one-dimensional (radial) nonstationary temperature distribution is calculated in the cylinders and the cylinders are coupled by the blood flows. The system of one-dimensional equations (9) and the balance equations for the blood (6), (7), (10) are solved. The heat exchange between arteries and veins is taken into account by introducing counterflow heat exchangers between separate parts of the body. In [16] this model was supplemented by the equations of gas exchange and was used to study the thermal systems of deep rivers.

The most detailed description of the temperature field in the human body results from the use of the model of [15]. Kuznetz models the body by 10 cylinders (head, trunk, legs, feet, arms, hands) with separate layers representing the bones, muscle, fat, and skin. In each cylinder the two-dimensional form of (11) is considered (depending on the radius r and angle φ). In the description of heat exchange with the blood, only capillary heat exchange is taken into account, so that the temperature of the arterial blood is taken to be the same everywhere and is determined from (12). The problem is numerically solved and has been applied to the study of the thermal states of cosmonauts (astronauts) working outside the space capsule in a space unit. The results were compared with experiment, and conditions where the two-dimensional treatment is necessary were pointed out.

Distributed parameter models are widely used in the mathematical modeling of cancer therapy, where the thermal states of separate regions of the body change. In [27-33] the temperature fields were calculated under absorption of electromagnetic radiation by the tissues; (11) was considered together with the electromagnetic field equations. In [34-36] the problem of calculating the states of freezing tissue was examined with the goal of choosing optimal methods of doing cryosurgical operations. Using the model (11) methods of regulating the amount of blood flow in different parts of the body according to their temperatures have been worked out [37, 38], and methods of calculating the process of restoration of skin temperature after local cooling [39, 40]. Equation (11) is the basis of many methods of measuring the thermophysical properties of living tissue [41-43] and also has been used to solve some methodological problems in temperature measurements in the body [44]. We note that in the above solutions, only heat-exchange processes in a limited part of the body were considered, and the temperature of the incoming blood and the boundary conditions were supplied from experimental data.

We now discuss multistage modeling methods. In the works cited so far, the thermal state of the entire organism was described using models without a great deal of detail;

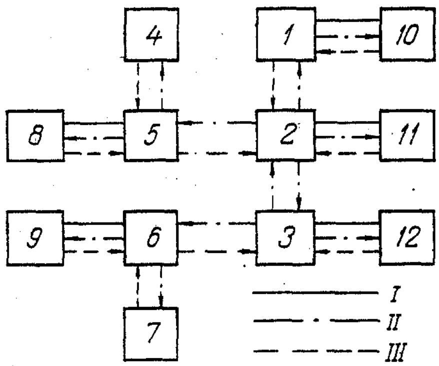

Fig. 2

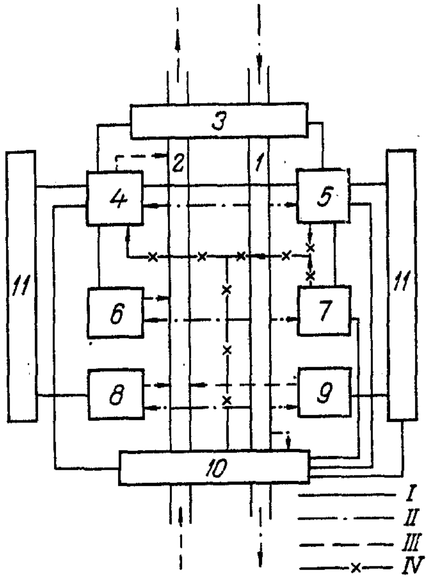

Fig. 3

Fig. 2. First stage model. 1) Head; 2) thoracic cavity; 3) peritoneal cavity; 4) hands; 5) arms; 6) legs; 7) feet; 8-12) membranes. I) thermal links; II) arterial blood flow; III) venous blood flow.

Fig. 3. Second stage model (thoracic cavity). 1) Aorta; 2) vena cava inferior; 3) diaphragm; 4) liver; 5) stomach; 6) gallbladder; 7) pancreas; 8, 9) right and left kidneys; 10) intestines; 11) membranes; I) thermal links; II) arterial blood low; III) venous blood flow; IV) portal vein blood.

lumped parameter models using averaged temperatures of different parts of the body, or one-dimensional distributed parameter models. A more detailed description was considered in [15] where two-dimensional temperature distributions were calculated for a set of cylinders. Models describing the thermal state of the organism as a whole along with the thermoregulatory system are used in the study of the human body under extreme climatic conditions, high physical stress, or in the design of survival systems.

In the study of problems involving spatial temperature distributions in separate parts of the body or organs, distributed parameter models are used having more detail, but considering only a part of the body isolated from the whole organism. This approach has been used to solve problems in cryosurgery [34-36], local hyperthermia [27-33], and hypothermia [39, 40], and thermographic diagnostics [45, 46].

There are problems requiring both a high degree of detail in the description of the thermal states of separate organs and the treatment of the thermal state of the organism as a whole, including thermoregulatory processes. This type of problem arises in the study of thermographic diagnostics of the pathology of the internal organs. The problem consists of establishing connections between the disturbance of physiological processes in the internal organs (changes in heat production or blood flow) and changes in the temperature field on the surface of the skin opposite to the corresponding organ. With the help of mathematical modeling, it is possible to examine hypotheses on the causes of changes in the temperature field for a particular pathology and thus to establish an objective criteria for thermographic diagnostics.

The problem requires a detailed description of heat-transport processes in the surface tissues in order to model the temperature field of the skin, and also the calculation of heat exchange in the internal organs for the temperature changes that occur for the pathology under study. In the analysis of the temperature field in a separated region of the body, the temperatures or heat fluxes from the surrounding parts must be given, as well as the temperature of the incoming blood flows. This information can only be obtained by preliminary calculation of heat-exchange processes in the entire body. In addition, the calculation of the temperature fields is complicated by the active thermoregulatory processes. The blood flow rates, the magnitudes of internal heat sources, the loss by evaporation are all determined by the state of the thermoregulatory system. In the feedback equations for the thermoregulatory system, the deviations of the temperatures of separate parts of the body from their reference values appear. Therefore, unlike the case of a passive system, without a preliminary analysis of the thermal state of the human body as a whole, it is impossible to specify the effect of other parts of the body on the part under study, as well as internal effects.

A mathematical model combining a detailed analysis of the temperature distribution and a description of the thermal state of the entire organism can be constructed using multistage methods [47]. The temperature field is calculated in several stages, where in each successive stage, the number of elements of the system being considered is decreased and the degree of detail is increased. The results at a particular stage are used to give the boundary conditions for the next stage; thus the interaction of the elements is taken into account.

We consider a possible multistage model for the solution of problems in thermographic diagnostics. We consider a model with three stages.

In the first stage, the thermal state of the entire organism is described with a lumped parameter model at the level involving the volume-averaged temperatures of separate parts of the body and the mean flow rate temperatures of blood flows coupling these parts together. The output parameters used in further calculations are the temperature of the blood going into the internal organ and into the peritoneal cavity membrane, and the temperature of the diaphragm and muscle bounding the peritoneal cavity. The first stage model also includes a mathematical description of the thermoregulatory system. The division of the body into elements for the lumped parameter model can be done in different ways. For the multistage approach we choose the model shown in Fig. 2.

In the second stage of the calculation, we determine the average temperatures of the separate organs of the peritoneal cavity and the temperatures of the blood flows between them. A schematic diagram illustrating the division into elements and the interaction between them is shown in Fig. 3. The most difficult feature of this stage of the model is the determination of the starting-point parameters such as the thermal conductivities and blood flow rates. These difficulties arise because the physiological information is not perfectly definite and also because of the complicated geometry of the system. In testing the applicability of the model, it is most important to examine the effect of errors in the starting-point data on the results of the calculation. After carrying out the second-stage calculation, the temperatures of the internal organs near the membrane are known, and thus we can now formulate the boundary conditions for the propagation of heat in the membrane.

In the third stage the problem of calculating the spatial temperature field inside the tissues of the peritoneal cavity membrane is studied. A distributed parameter model is used and (11) is solved inside a bounded sector of a complete cylinder. The problem is solved numerically, and thus nonuniformities of the anatomical structure can be taken into account, as well as the nonuniform distribution of capillary blood flow in the membrane layers. The previous stages of the calculation are used to specify boundary conditions on the inner surfaces of the peritoneal walls, and the temperature and flow rate of blood in different membrane layers. After solution of the third stage, we obtain the temperature distribution on the surface of the body, and this can be compared with the thermographic data.

The multistage approach allows one to consider models of the temperature field with a great deal of detail, but not requiring complicated computer programs of each specific problem. The algorithm for the multistage model discussed above consists of a program for the generalized lumped parameter model (5)-(8) and a program for the numerical solution of (11) inside a cylinder.

NOTATION

T_i, T_{ai}, T_{vi}, volume-averaged temperatures of tissue, arterial blood, and venous blood of the i-th element; T_{ai}^{in}, T_{vi}^{in}, T_{ai}^{out}, T_{vi}^{out}, mean flow rate temperature of incoming (in) and outgoing (out) arterial and venous blood flows of the i-th element; Q_{ij}, conductive heat flux; Q_{ai}, Q_{vi}, Q_{ki}, heat fluxes from blood to tissue transferred in arteries, veins, and capillaries; Q_{Mi}, output of internal heat sources; Q_{ei}, heat flux into the external medium due to evaporation; σ_{ij}, thermal conductivity tensor; σ_{ai}, σ_{vi}, σ_{av}, thermal conductivities between tissues and arteries, tissues and veins, and arteries and veins; C_i, total heat capacity of the i-th element; c_{kb}, specific heat capacity of the blood; G_{ai}^{in}, G_{vi}^{in}, G_{ki}, G_{aij}, G_{vij}, mass flow rates of arteries, veins, and capillaries entering the i-th element; mass flow rates of arteries and veins from the j-th element to i-th; G_{Σ}, total mass flow rate in the blood circulation system; m_{ai}, m_{vi}, mass of the arterial and venous blood of the i-th element; T(x), temperature field of the tissue in the quasihomogeneous body model; λ, cρ, effective thermal conductivity and specific heat of the tissue; G, mass flow rate of capillary blood per unit volume; α_a, α_v, volumetric heat-transfer coefficients from tissues to arterial and venous blood; q_M, specific output of internal heat sources.

LITERATURE CITED

1. L. T. Fan, F. T. Hsu, and C. L. Hwang, "A review of mathematical models of the human thermal system," IEEE Trans. Bio.-Med. Eng., 18, No. 3, 218-234 (1971).
2. C. L. Hwang and S. A. Konz, "Engineering models of the human thermoregulatory system. A review," IEEE Trans. Bio.-Med. Eng., 24, No. 4, 309-325 (1977).
3. A. F. Emery, "The numerical simulation of the thermal response of man," 6th Int. Heat Transf. Conf., Toronto, 1978. Keynote paper, Vol. 6. 483-494, Ottowa (1978).
4. J. A. J. Stolwijk and J. D. Hardy, "Temperature regulation in man — a theoretical study," Pflugers Arch., 291, 129-162 (1966).
5. J. A. J. Stolwijk, "A mathematical model of physiological temperature regulation in man," NASA, CR-1855, Washington (1971).
6. Emery, Short, Hay, Krayning, and Lee, "Numerical thermal modeling of the human body under physical exercise or under nonionizing electromagnetic radiator," Teploperedacha, 98, No. 2, 146-155 (1976).
7. I. I. Ermakova, "Study of dynamical processes in the thermoregulatory system of man by the methods of numerical modeling," Doctoral Dissertation, Biological Sciences, Kiev (1974).
8. T. A. Tsivina and A. N. Azhaev, "A model of heat exchange in man and the evaluation of its parameters," Fiz. Cheloveka, 5, No. 1, 159-166 (1979).
9. E. H. Wissler, "A mathematical model of the human thermal system," Bull. Math. Biophys., 26, 147-166 (1964).
10. E. Zanchini, M. Mariotti, and S. Salvini, "Heat transfer in the circulatory system," Rassegna Bioingegneria, No. 8, 33-44 (1979).
11. J. C. Chato, "Heat transfer to blood vessels," Trans. ASME, J. Biomech. Eng., 102, No. 2, 110-118 (1980).
12. M. M. Korotkevich, E. V. Maistrakh, N. V. Pilipenko, and A. V. Sigalov, "Mathematical modeling and experimental study of the temperature field on the surface of the human body," in: Reports on the All-Union Conference on Thermography in Medicine and Its Applications [in Russian], Leningrad (1982), pp. 118-119.
13. V. Ferro and M. Tilippi, "L'equazione generale del la propagazione termica nelle applicazioni biomediche," Termotecnica, 32, No. 9, 487-490 (1978).
14. G. R. Gordon, R. B. Roemer, and S. M. Horvath, "A mathematical model of the human temperature regulatory system — Transient cold exposure response," IEEE Trans. Biomed. Eng., 23, No. 6, 434-444 (1976).
15. L. H. Kuznetz, "A two-dimensional transient mathematical model of human thermoregulation," Am. J. Physiol., 237, No. 5, 266-277 (1979).
16. E. H. Wissler, "Simulation of thermal transients during deep diving," Proc. Summer Comput. Simul. Conf., pp. 477-488 Chicago, Ill. (1977); Calif. (1977).
17. T. H. Bezinger, "Role of thermoreceptors in thermoregulation," Proc. Int. Symp. Stockholm, pp. 379-397 (1976).
18. J. Bligh, "Thermoregulation: what is regulated and how," Proc. Int. Union Physiol. Sci. 27th Int. Congr., Paris, Vol. 12, pp. 22-23 (1977).
19. I. I. Ermakova, "Functional roles of central and peripheral thermosensors in regulation," in: Cybernetic Physiology — Proc. of the 1st Int. Conf. on Cyber. Physiol., Moscow (1981), pp. 63-64.
20. I. I. Ermakova, "On the effect of mean skin temperatures on thermoregulatory processes," in: Mathematical Modeling and Experimental Study of Physiological Systems [in Russian] Kiev (1978), pp. 49-58.
21. E. H. Wyndham, "The role of core and skin temperatures in man's temperature regulation," J. Appl. Physiol., 20, 31-36 (1965).
22. P. A. Hamcock, "Simulated and experimental temperature response in man during exercise in varying environments," Comput. Biol. Med., 10, No. 1, 1-9 (1980).

23. J. Werner, "Mathematical treatment of structure and function of the human thermoregulatory system," Biol. Cybern., 25, 93-101 (1977).
24. V. A. Fukina and A. I. Zhavoronkov, "Mathematical model of the thermal state of man in heating clothing," in: Garment Industry Technology [in Russian], Moscow, pp. 74-77.
25. O. V. Korobko, V. N. Papkovich, and S. B. Minkin, "Mathematical analysis of heat-exchange processes in biological systems under general thermal stimulation," in: Heat and Mass Transfer Processes and Techniques [in Russian], Nauka i Tekhnika, Minsk (1978), pp. 15-17.
26. O. V. Korobko and T. L. Perel'man, "Temperature distribution in the human body under hyperthermia," Inzh.-Fiz. Zh., 26, No. 3, 523-528 (1974).
27. F. Bardati, G. Gerosa, and P. Lampariello, "Temperature distribution in electromagnetically irridated cylinders simulating living tissues," 14th Int. Symp. Appl. Eng., Monaco (1979); Paris (1979), pp. 189-191.
28. J. Eichler, J. Knof, and H. Lenz, "Temperature distribution in tissue during laser irradiation," Rad. Envirn. Biophys., 15, No. 3, 277-287 (1978).
29. B. S. Guru and K. M. Chen. "Hyperthermia by local EM heating and local conductivity change," IEEE Trans. Biomed. Eng., 24, No. 5, 473-477 (1977).
30. R. K. Jain, "Transient temperature distribution in an infinite perfused medium due to a time-dependent spherical heat source," Trans. ASME J. Biomech. Eng., 101, No. 1, 82-86 (1979).
31. H. N. Kriticos and H. P. Schwan. "Potential temperature rise induced by an electromagnetic field in brain tissues," IEEE Trans. Biomed. Eng., 26, No. 1, 29-34 (1979).
32. L. A. Priebe and A. J. Welch, "A dimensionless model for the calculation of the temperature increase in biological tissues exposed to nonionizing radiation," IEEE Trans. Biomed. Eng., 26, No. 4, 244-250 (1979).
33. Emery, Kramer, Hay, and Lin, "Microwave heating of a rabbit eye in the study of cataracts," Teploperedacha, 97, No. 1, 127-132 (1975).
34. Komini, Del, and Jiudiche, "Thermal aspects of cryosurgery," Teploperedacha, 99, No. 1, 8-16 (1977).
35. Rubinski and Schitzer, "Stefan problem for biological tissue surrounding a cryogenic probe," Teploperedacha, 98, No. 3, 187-192 (1976).
36. Cooper and Tresek, "Freezing of biological tissue," Teploperedacha, 94, No. 2, 137-139 (1972).
37. Eberkart, Jackson, and Tresek, "Application of heat-transport models in the determination of local blood flow," Teploperedacha, 95, No. 1, 123-130 (1973).
38. J. E. Francis, R. Roggli, T. J. Love, and C. P. Robinson, "Thermography as a means of blood perfusion measurement, "Trans. ASME J. Biomech. Eng., 101, No. 4, 246-249 (1979).
39. P. P. Rao, J. E. Francis, and T. J. Love, "Transient thermal response of the thorax due to provocative cooling," Proc. 6th Can. Conf. Appl. Mach., Vol. 2, pp. 923-924, Vancouver (1977).
40. J. Steketee and M. J. Van der Hock, "Thermal recovery of the skin after cooling," Phys. Med. Bio., 24, No. 3, 583-592 (1979).
41. T. A. Balasubramanian and H. F. Bowman, "Thermal conductivity and thermal diffusivity of biomaterials: a simultaneous measurement technique," Trans. ASME J. Biomech. Eng., 99, No. 3, 148-154 (1977).
42. H. F. Bowman, T. A. Balasubramanian, A. Mak, and M. Ibhahim, "A simulation model to investigate the influence of blood vessels on tissue thermal property measurements," Proc. 29th Annual Conf. Eng. Med. and Biol., Vol. 18, Boston (1976), p. 311.
43. Cooper and Tresek, "Probe for determining the thermal conductivity of biological tissue," Teploperedacha, 94, No. 2, 5-14 (1972).
44. S. D. Mahanty and R. B. Roemer, "The effect of pressure on skin temperature measurements for a disk sensor," Trans. ASME J. Biomech. Eng., 101, No. 4, 261-266 (1979).
45. V. N. Koshelev and O. L. Pertsov, "Estimate of the vertical heat transport in membranes of the human body" [in Russian], in: Thermography Conf. Scient. Trans. Issue 2, Moscow (1978), pp. 197-208.
46. V. N. Koshelev, "Mathematical modeling of temperature—time relations in membranes of the human body," in: Reports of the All-Union Conf. on Thermography and Its Application [in Russian], Leningrad, pp. 116-117 (1982).
47. G. N. Dul'nev and A. V. Sigalov, "Multistage modeling of the thermal states of complex systems," Inzh.-Fiz. Zh., 45, No. 4, 651-656 (1983).